Stability of Longest-Queue-First Scheduling in Linear Wireless Networks with Multihop Traffic and One-Hop Interference*

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Abstract— **We consider the stability of the longest-queue-first (LQF) scheduling policy in wireless networks with multihop traffic under the one-hop interference model. Although it is well known that the back-pressure algorithm achieves the maximal stability, its computational complexity is very high. In this paper, we are interested in LQF, a low-complexity scheduling algorithm, which has been shown to have near optimal throughput performance in many networks with single-hop traffic flows. In this paper, we are interested in the performance of LQF for multihop traffic flows. In this scenario, the analysis of local-pooling factors for LQF does not carry through because of the complicated coupling between queues due to multihop traffic flows. Using fluid limit techniques, we show that LQF achieves the maximal stability for linear networks with multihop traffic and a single destination under the one-hop interference.**

I. INTRODUCTION

The scheduling problem in wireless networks with multihop traffic has gained significant attention over the last few decades. One fundamental goal of the design of scheduling policies, among many others, is to decide the set of scheduled links at each time slot in accordance with the underlying interference model, such that the system is stable. The back-pressure algorithm has been proved to be throughput optimal for general multihop traffic settings [1]; i.e., it stabilizes the network as long as the arrivals are within the network throughput region. The algorithm, however, requires the network to solve a maximum-weight independent set problem at each time instance and requires the nodes to exchange queue lengths with their neighbors constantly.

In this paper, we study the stability of longest queue first (LQF) scheduling, which selects links according to queue lengths in a greedy fashion. LQF has been extensively studied as a low complexity approximation of MaxWeight scheduling, and has great throughput and delay performance in many networks. The conditions under which LQF is throughput optimal has been established by Dimakis and Walrand [2] and the performance guarantee of LQF in general networks has been characterized by Joo et al. [3] and estimated under different scenarios [3]–[6]. However, these results all assume single-hop traffic flows in the networks. For networks with multihop traffic, transmitted packets at one link may become the *internal* arrivals to another link. Hence links with small queues may affect the ones with large queues by providing internal arrival, which makes it difficult to analyze the system using local-pooling factors since the links with larger queues are no longer isolated from those with smaller queues. Although Brzezinski et al. [7] developed conditions for networks with multihop traffic under which a back-pressure-based greedy algorithm achieves the maximal throughput, the performance of LQF for networks with multihop traffic flows is still open. We are interested in tackling this problem.

This paper proves the throughput optimality of LQF in a simple network, i.e., a linear network (also known as a tandem network [8]) with single destination and one-hop interference model (also known as primary or node-exclusive interference model). While the result is only for linear networks, it is the first step to understand the following question: to achieve throughput optimality in a wireless network with multihop traffic flows that have fixed routes, is it sufficient to use queue lengths as weights instead of using differential queues? If the answer is positive, then nodes do not need to constantly exchange queue lengths, which eliminates a significant amount of communication overhead.

The novelty in this paper lies in the techniques we adopt to show the stability of the fluid model after the standard construction of fluid limits. Instead of using an explicit Lyapunov function, we follow the observations from the simulation trajectories of an example network and examine the evolution of the states of the deterministic fluid limits. We first show that the system will eventually stay in the state where the first fluid is zero. Then by combining the first two fluids into one using a coupled network argument, we reduce the size of the network by one and conclude that all fluids eventually become zero by induction.

The paper is organized as follows. We introduce the basic model in Section II. In Section III we present our result of throughput optimality of LQF, as well as an intuitive example, formal notations and network equations, construction of fluid limits, and the proof by transient states and coupled network arguments. Section IV concludes the paper.

II. MODEL

Consider a linear network represented by a directed graph $G = (V, L)$ with $|V| = N + 1$ nodes and $|L| = N$ links as shown in Fig. 1. Let $V = \{v_1, v_2, \dots, v_{N+1}\}$ and $L = \{l_1, l_2, \dots, l_N\}$, where l_i is the link from node v_i to node v_{i+1}. We assume v_i is the origin node of flow f_i with exogenous (or external) packet arrival rate α_i for $1 \le i \le N$, and all flows have the same destination v_{N+1}. In the paper

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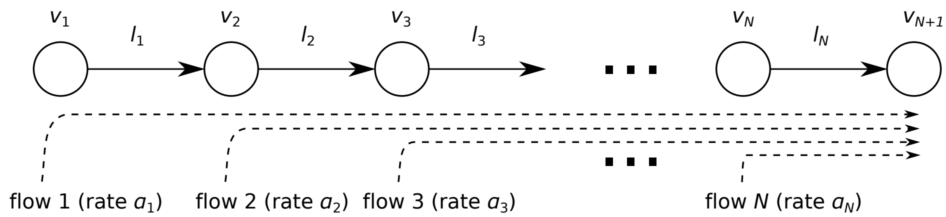

Fig. 1. A linear network with N links. The i^{th} dashed line indicates the flow with source node v_i and destination node v_{N+1} and exogenous packet arrival rate α_i.

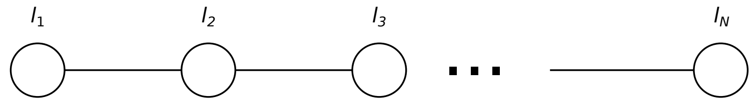

Fig. 2. The one-hop interference graph of Fig. 1

we focus on one-hop interference model, so the interference graph is as shown in Fig. 2.

We assume time is slotted, and in each time slot a subset of the links can be scheduled. Once scheduled, a packet at link l_i is transmitted from node v_i to node v_{i+1} and join the queue at node v_{i+1} if it has not reached the destination v_{N+1}, or leave the network otherwise. As a result, besides external packet arrivals, there can also be *internal* packet arrivals to a node according to the schedule of other links.

The scheduler decides a subset of the links $s \subseteq L$ to be activated in every time slot, called a schedule, such that the schedule is feasible (no interference between scheduled links) and maximal (no other link can be added to the schedule), and then the queue length at each transmitter in the activated subset reduces by 1 if there are any packets to schedule, or remain 0 otherwise. The schedule (also known as activation set) s is represented by an *activation vector* m, which is a binary column vector with N elements. According to the interference model shown in Fig. 2, a schedule s is feasible if no two adjacent links are activated at the same time; i.e., the activation vector m does not contain two consecutive 1's.

In the paper we are interested in LQF with arbitrary tie-breaking rules, and we define it as follows. At each time slot, let Z_i be the queue length at link l_i for $1 \leq i \leq N$. The set of links are sorted with arbitrary tie-breaks such that $Z_{\sigma_1} \geq Z_{\sigma_2} \geq \cdots \geq Z_{\sigma_N}$, where $(\sigma_1, \sigma_2, \ldots, \sigma_N)$ is the sorted index vector. LQF starts with the schedule $\mathcal{E} = \{\sigma_1\}$, and proceed to consider $i = 2, 3, \ldots, N$ inductively and append σ_i to $\mathcal{E}$ if σ_i does not interfere with any link that is already in $\mathcal{E}$. This procedure ends after the link l_{σ_N} is considered and the resulting schedule $\mathcal{E}$ is the schedule chosen by LQF.

III. STABILITY

In this section we analyze the stability property of LQF in the linear network under the one-hop interference model. We first state the main theorem with the proof outline and an illustrative example, and then proceed with the formal proof.

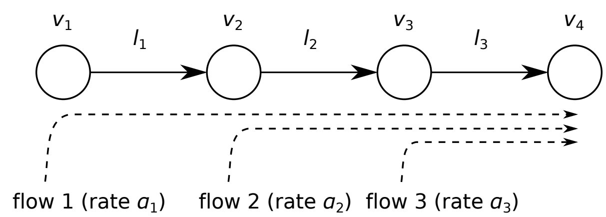

Fig. 3. A three-link linear network

A. Main Result

Theorem 1: LQF is throughput optimal on linear networks with the single-destination multihop traffic under the one-hop interference. ◇

Theorem 1 states that LQF can stabilize a linear multihop traffic network. Thus using queue lengths instead of queue differences is sufficient. This result may also shed light on the throughput performance of LQF in other networks with multihop traffic, in which the routes are fixed.

The proof consists of the following steps. We first follow the standard construction of the fluid limits. Then we show that eventually the fluids should be such that each fluid is less than or equal to at least one neighbor fluid; i.e., no fluid dominates all its neighbors. After that we prove that the first fluid must decrease with rate at least $\epsilon > 0$. Finally we use a coupled network argument to show that all fluids eventually go to zero under admissible arrival rates, which implies throughput optimality.

We next demonstrate the key ideas of the proof using an example.

B. Three-Link Linear Network

We consider the simple linear network example with four nodes $\{v_1, v_2, v_3, v_4\}$ and three links $\{l_1, l_2, l_3\}$ as shown in Fig. 3. Suppose flow i has origin v_i and destination v_4 with Bernoulli arrival of rate α_i for $i = 1, 2, 3$. The interference is such that two adjacent links cannot be scheduled at the same time, so either $\{l_1, l_3\}$ or $\{l_2\}$ is scheduled in each time slot. Let $Z_i(n)$ be the queue length on link l_i at time slot n. Then at each time slot, the LQF scheduler first selects

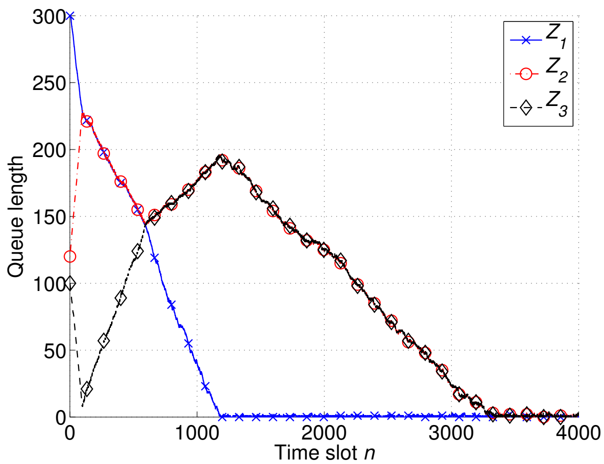

Fig. 4. Queue evolution of the three-link linear network under LQF

the longest queue with arbitrary tie-breaking, then append it to either $\{l_1, l_3\}$ or $\{l_2\}$ according to the first select.

A typical queue evolution graph for the three-link linear network under LQF is shown in Fig. 4. Here the initial queue lengths are $Z_1(0) = 300$, $Z_2(0) = 120$ and $Z_3(0) = 100$, with arrival rates $\alpha_1 = 0.25$, $\alpha_2 = 0.1$ and $\alpha_3 = 0.05$. We make several interesting observations from the figure:

1) The queue lengths look like piecewise linear functions (this is partially due to the law of large numbers over the arrival process).
2) The queue dynamics are somewhat complex at the beginning of the time slots (largely due to the internal arrival from other links).
3) The first queue eventually drop to close to zero, and the behavior of the rest queues become more predictable.
4) Finally all queues seem to be close to zero, so the system is expected to be stable.

In light of the above findings, we first claim that after some time we have either $Z_1(t) = Z_2(t) \geq Z_3(t)$ or $Z_1(t) \leq Z_2(t) = Z_3(t)$, since otherwise one queue will be larger than all its neighbors, resulting a decreasing difference with its neighbors under LQF. We can then see that if $Z_1(t)$ and $Z_2(t)$ stick together then they must both decrease since $2\alpha_1 + \alpha_2 = 0.6 < 1$, and if $Z_2(t)$ and $Z_3(t)$ stick together then we can compute that $Z_1(t)$ must decrease with rate $\frac{1}{2} - \alpha_1 - \frac{1}{4}\alpha_2 + \frac{1}{4}\alpha_3 = 0.2375 > 0$ since the service rates on links l_1 and l_3 must be equal. We also argue that when the first queue drops to close to zero, it cannot rise again since if it did it would be "forced back" immediately. So at last the three-link linear network is reduced to a 2-link linear network and the remaining two queues go to close to zero as well. The above intuition will lead our way to the rigorous proof for the general linear network case in the rest of this paper.

C. Notations and Network Equations

We use the following notations:

- R: the $(N+1)$-by-N routing matrix as is defined by Tassiulas and Ephremides [1], where $R_{ik} = -1$ if link l_k goes from node v_i, $R_{ik} = 1$ if link l_k goes to node v_i with $i \neq N+1$, and $R_{ik} = 0$ otherwise, for $1 \leq i \leq N+1$ and $1 \leq k \leq N$. Then in the linear network case the routing matrix is given by

$$R = \begin{pmatrix} -1 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & \ddots & \ddots & \vdots \\ 0 & 1 & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 & -1 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}, \qquad (1)$$

where the last row is all-zero since the node v_{N+1} is the destination for all flows.
- M: the N-by-r binary-entry matrix whose columns are the activation vectors of the possible maximal schedules, where r is the total number of possible maximal schedules. By a little abuse of notation we regard the columns of M as the elements of the set $M = \{m_1, m_2, \ldots, m_r\}$.
- $Z_i(n)$ for $1 \leq i \leq N$: the queue length at link l_i at time slot n (before arrivals and departures happen in time slot n).
- $E_i(n)$ for $1 \leq i \leq N$: the cumulative exogenous arrival to link l_i up to time slot n. We assume the increments of $(E_i(n))$ are temporally i.i.d. and independent across i. The exogenous arrival rate is $\mathbb{E}[E_i(n) - E_i(n-1)] = \alpha_i$ for all n.
- $A_i(n)$ for $1 \leq i \leq N$: the cumulative arrival to link l_i up to time slot n. This includes both exogenous and internal arrivals.
- $D_i(n)$ for $1 \leq i \leq N$: the actual cumulative departure from link l_i up to time slot n.
- $T_j(n)$ for $1 \leq j \leq r$: the cumulative service time (in number of time slots) of schedule m_j up to time slot n.
- $Y_i(n)$ for $1 \leq j \leq r$: the cumulative idle time (in number of time slots) of link l_i up to time slot n (when link l_i is chosen by the scheduler but does not actually send packets). Note that even if the queue at link l_i is empty at the time of scheduling, the scheduler can still choose the schedule $m \in M$ such that $l_i \in m$, in which case $Y_i(n)$ will increase instead of $D_i(n)$. For non-idling (or work-conserving) scheduling policies $Y_i(n)$ can only increase when the queue at link l_i is empty.

Let $Z(n), E(n), A(n), D(n), T(n), Y(n)$ be the corresponding column vectors. Then we refer to $\mathbb{X}(n) = (Z(n), E(n), A(n), D(n), T(n), Y(n))$ as the *queueing network process*. Let $\mathcal{X} = \mathbb{Z}_+^{5N+r}$ be the space where $\mathbb{X}$ lives. Then $\mathbb{X}$ is an $\mathcal{X}$-valued stochastic process defined for nonnegative integer values of n. Let Ω be the set of sample paths specifying the exogenous arrival processes $(E_i(n))$ and the possible tie-breaks of the scheduler. Note that under the LQF policy $\mathbb{X}(\cdot)$ forms a discrete Markov chain. The dynamics of the network are described by the following *queueing network equations*:

$$A(n) = E(n) + (R_0 + I_N)D(n-1) \quad (2)$$

$$Z(n) = Z(0) + A(n) - D(n) \quad (3)$$

$$\sum_{j=1}^{r} T_j(n) = n \quad \big(\text{or } e^{\mathrm{T}}T(n) = n\big) \quad (4)$$

$$D(n) = MT(n) - Y(n) \quad (5)$$

for any nonnegative integer n, where $(\cdot)^{\mathrm{T}}$ denotes the transpose, e is the all-one column vector, R_0 is the square matrix consisting of the first N rows of the routing matrix R, and I_N is the N-by-N identity matrix. Moreover, if the scheduling is non-idling, then we also have

$$Y_i(n) - Y_i(n-1) = \begin{cases} 1 & \text{if } Z_i(n-1) = 0 \text{ and} \\ & \displaystyle\sum_{j:\, i \in m_j} (T_j(n) - T_j(n-1)) = 1 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

for $1 \leq i \leq N$ and $n \geq 1$. All of the variables take nonnegative integers in each component, and E, A, D, T, Y are nondecreasing. Also we assume the initial conditions are

$$E(0) = A(0) = D(0) = Y(0) = 0 \text{ and } T(0) = 0. \quad (7)$$

For the LQF policy, we have in addition to (2), (3), (4), (5), (6) and (7):

$$T_j(n) - T_j(n-1) = 1 \Rightarrow m_j \in \text{LQF}(Z(n-1)), \quad (8)$$

where LQF(Z) is the set of possible LQF maximal schedules given queue length vector Z. We assume that the schedule is always maximal regardless of the queues being empty or not, so LQF$(Z) \subseteq M = \{m_1, m_2, \ldots, m_r\}$.

D. Fluid Limits

We define the scaled systems based on the queueing network process for each sample path, and show that the scaled systems converge along some subsequence to deterministic systems called fluid limits.

We first extend the definition of $\mathbb{X}$ for arbitrary nonnegative time $t \geq 0$ by piecewise linear interpolation

$$\mathbb{X}(t) = (1 + \lfloor t \rfloor - t)\mathbb{X}(\lfloor t \rfloor) + (t - \lfloor t \rfloor)\mathbb{X}(\lfloor t \rfloor + 1),$$

where $\lfloor t \rfloor$ is the largest integer less than or equal to t. Then $\mathbb{X}$ is an $\bar{\mathcal{X}}$-valued stochastic process with $\bar{\mathcal{X}} = \mathbb{R}_+^{5N+r}$, and is continuous for $t \geq 0$ given any fixed sample path $\omega \in \Omega$.

Let $|\cdot|$ be the L^1-norm of $\mathcal{X}$. Fix $\omega \in \Omega$, and let $\mathbb{X}^x(t)$ be the queueing network process with initial state $\mathbb{X}(0) = x$ for $x \in \mathcal{X}^0 = \{y \in \mathcal{X} \mid |y| > 0\}$ and define the *scaled system*

$$\bar{\mathbb{X}}^x(t) = \frac{1}{|x|}\mathbb{X}^x(|x|t).$$

We then have the following proposition giving the existence of the fluid limits, which is similar to Theorem 4.1 in the paper by Dai [9]. We denote by $\mathbb{Z}_+$ the set of positive integers, and use subscripts to indicate the indices of sequences.

Proposition 1: For a work-conserving scheduling policy, for almost any sample path $\omega \in \Omega$ and any sequence of initial states $(x_k)_k$ with $\{x_k \mid k \in \mathbb{Z}_+\} \subseteq \mathcal{X}^0$ and $|x_k| \to \infty$ as $k \to \infty$, there exists a subsequence $(k_p)_p$ with $|x_{k_p}| \to \infty$ as $p \to \infty$ such that

$$\bar{\mathbb{X}}^{x_{k_p}}(0) \to \bar{\mathbb{X}}(0) \quad \text{as } p \to \infty$$

and

$$\bar{\mathbb{X}}^{x_{k_p}}(t) \to \bar{\mathbb{X}}(t) \quad \text{u.o.c. as } p \to \infty$$

for some $\bar{\mathbb{X}}\colon \mathbb{R}_+ \to \bar{\mathcal{X}}$, where "u.o.c." stands for uniform convergence over compact sets [10]. Furthermore,

$$\bar{A}(0) = \bar{D}(0) = \bar{Y}(0) = 0 \text{ and } \bar{T}(0) = 0 \quad (9)$$

$$\bar{A}(t) = \alpha t + (R_0 + I_N)\bar{D}(t) \quad (10)$$

$$\bar{Z}(t) = \bar{Z}(0) + \bar{A}(t) - \bar{D}(t) \quad (11)$$

$$e^{\mathrm{T}}\bar{T}(t) = t \quad (12)$$

$$\bar{D}(t) = M\bar{T}(t) - \bar{Y}(t) \quad (13)$$

and

$$\int_0^\infty \bar{Z}_i(t)\mathrm{d}\bar{Y}_i = 0 \quad i = 1, 2, \ldots, N. \quad (14)$$

Moreover, all components of $\bar{\mathbb{X}}$ are absolutely continuous [10] because they are Lipschitz continuous, and $\bar{A}, \bar{D}, \bar{T}, \bar{Y}$ are nondecreasing.

Particularly, the fluid limits under LQF satisfy

$$\frac{\mathrm{d}\bar{T}_j}{\mathrm{d}t}(t) > 0 \Rightarrow m_j \in \text{LQF}(\bar{Z}(t)) \quad j = 1, 2, \ldots, r, \quad (15)$$

where LQF$(\bar{Z})$ is the set of LQF schedules for links $\{i \mid \bar{Z}_i > 0\}$, and t is assumed regular so the derivatives exist.$\diamond$

Remark. Basically, (9) is the initial condition assumption. (10) says the arrival rates consist of exogenous part and internal part. (11) is the queue evolution equation. (12) comes from the fact that in each time slot there is exactly one maximal schedule chosen by the scheduler. (13) gives the relation among departures, serving time of schedules and idling time. (14) means a link can be idle (when it is chosen by the scheduler) only if the queue length at the link is 0. (15) states that only maximal schedules satisfying the LQF property given the queue fluids $\bar{Z}(t)$ can be chosen at time t, but it does not specify the fractions of the schedules that LQF could choose. The proof of Proposition 1 is in Appendix A.

E. Transient States with Dominating Fluids

We first identify a set of transient states of the space of the queue fluid vectors. Let

$$B_1 = \{Z \in \mathbb{R}_+^N \mid Z_1 > Z_2\}$$

$$B_2 = \{Z \in \mathbb{R}_+^N \mid Z_2 > Z_1, Z_2 > Z_3\}$$

$$\vdots$$

$$B_{N-1} = \{Z \in \mathbb{R}_+^N \mid Z_{N-1} > Z_{N-2}, Z_{N-1} > Z_N\}$$

$$B_N = \{Z \in \mathbb{R}_+^N \mid Z_N > Z_{N-1}\}$$

and let

$$B = \bigcup_{i=1}^{N} B_i.$$

So B is the set of queue fluid vectors such that some queue strictly dominates all of its neighbors (one link has at most two neighbors in linear networks), while $\mathbb{R}_+^N \backslash B$ is the set of queue length vectors without any queue strictly dominating all of its neighbors. We then have the following lemma. (For convenience we ignore all the bars over the fluid limit processes in the rest of the paper.)

Lemma 1: B is transient. Formally, given $\alpha_i < 1$ for any $i \in \{1, 2, \ldots, N\}$, for any initial conditions $Z(t_0) \in \mathbb{R}_+^N$ at time t_0 with $Z_{\max} = \max_i Z_i(t_0)$, there exists $c_1 > 0$ such that $Z(t) \notin B$ for any $t \geq t_0 + Z_{\max} c_1$ under LQF. ◇

Remark. The outline of the proof is as follows, and the proof can be found in Appendix B.

1) If $Z \in B$, then there are no adjacent dominating nodes.
2) Each dominating node loses its domination in time $Z_{\max}/(1 - \max_i \alpha_i)$.
3) Once a node loses domination, it cannot regain it.

F. Stability of the First Fluid Z_1

We now further divide $\mathbb{R}_+^N \backslash B$ into several partitions:

$$C_0 = \{Z \in \mathbb{R}_+^N \backslash B \mid Z_1 = 0\}$$

$$C_1 = \{Z \in \mathbb{R}_+^N \backslash B \mid 0 < Z_1 = Z_2\}$$

$$C_2 = \{Z \in \mathbb{R}_+^N \backslash B \mid 0 < Z_1 < Z_2 = Z_3\}$$

$$\vdots$$

$$C_{N-1} = \{Z \in \mathbb{R}_+^N \backslash B \mid 0 < Z_1 < \cdots < Z_{N-1} = Z_N\}.$$

Then $\{C_0, C_1, \ldots, C_{N-1}, B\}$ forms a partition of $\mathbb{R}_+^N$. We then use the following two lemmas to show C_1, C_2, $\ldots$, C_{N-1} are all transient under admissible arrival rates, so the system has to eventually go to state C_0 where Z_1 stays at 0.

Lemma 2: If the arrival rate vector α is admissible, then there exists $\epsilon > 0$ such that for any regular time $t_1 \geq Z_{\max} c_1$ and $Z(t_1) \notin C_0$ we have

$$\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) \leq -\epsilon,$$

where $Z_{\max}$ and c_1 are given in Lemma 1. ◇

Remark. The idea of the proof is that for any sufficiently large regular time t_1 we show that if the fluid of the first queue is positive, then it must decrease with lower-bounded rate. Hence the first fluid reaches zero eventually.

Proof: Take $t_0 = 0$ in Lemma 1, and then we have $Z(t) \notin B$ for any $t \geq Z_{\max} c_1$, where $Z_{\max} = \max_i Z_i(0)$ and $c_1 = 1/(1 - \max_i \alpha_i)$. We let

$$W_1(t) = Z_1(t)$$

$$W_2(t) = Z_2(t) - Z_1(t)$$

$$\vdots$$

$$W_N(t) = Z_N(t) - Z_{N-1}(t)$$

and

$$J_0(t) = \{j \mid W_j(t) = 0\}.$$

Then $Z(t) \in \mathbb{R}_+^N \backslash B$ implies $J_0(t) \neq \emptyset$. For regular time t, we further let

$$J(t) = \left\{ j \in J_0(t) \;\middle|\; \frac{\mathrm{d}W_j}{\mathrm{d}t}(t) = 0 \right\}.$$

We claim that $J(t) \neq \emptyset$. To show this claim by contradiction, we suppose $J(t) = \emptyset$. We make the following observations:

1) We first notice that for $j_1 = \min\{j | j \in J_0(t)\}$ we must have $\frac{\mathrm{d}}{\mathrm{d}t} W_{j_1}(t) > 0$. If this is not the case, we would have $\frac{\mathrm{d}}{\mathrm{d}t} W_{j_1}(t) < 0$ and $j_1 \geq 2$. Then it would follow that there exists some $\delta > 0$ such that for any $s \in (t, t + \delta)$ we have $Z_1(s) > Z_2(s)$ if $j_1 = 2$, and $Z_{j_1 - 1}(s) > \max\{Z_{j_1 - 2}(s), Z_{j_1}(s)\}$ if $j_1 > 2$, which implies $Z(s) \in B$, a contradiction.
2) We then conclude that if all $j \in \{1, 2, \ldots, k\} \cap J_0(t)$ satisfies
$$\frac{\mathrm{d}}{\mathrm{d}t} W_j(t) > 0,$$
then either $k + 1 \notin J_0(t)$ or
$$\frac{\mathrm{d}}{\mathrm{d}t} W_{k+1}(t) > 0.$$
If this is not the case, there would exist $\delta > 0$ such that for any $s \in (t, t + \delta)$, we have $Z_k(s) > \max\{Z_{k-1}(s), Z_{k+1}(s)\}$ if $W_k(t) \geq 0$, or $Z_j(s) > \max\{Z_{j-1}(s), Z_{j+1}(s)\}$ for some $j < k$ otherwise, either of which leads to contradiction.
3) By induction we have $\frac{\mathrm{d}}{\mathrm{d}t} W_j(t) > 0$ for all $j \in J_0(t)$, which also leads to contradiction since by letting $j_2 = \max\{j | j \in J_0(t)\}$ there exists $\delta > 0$ such that for any $s \in (t, t + \delta)$ we have $Z_N(s) > Z_{N-1}(s)$ if $j_2 = N$, and $Z_{j_2}(s) > \max\{Z_{j_2 - 1}(s), Z_{j_2 + 1}(s)\}$ if $j_2 \neq N$. Then $Z(s) \in B$, which is a contradiction.

Hence the claim that $J(t) \neq \emptyset$ has been proved.

Now we fix a regular time $t_1 \geq Z_{\max} c_1$ with $Z_1(t_1) > 0$ and let

$$u = \min_{j \in J(t_1)} j.$$

Then $u \geq 2$ and $W_u(t_1) = \frac{\mathrm{d}}{\mathrm{d}t} W_u(t_1) = 0$; i.e., $Z_u(t_1) = Z_{u-1}(t_1)$ and $\frac{\mathrm{d}}{\mathrm{d}t} Z_u(t_1) = \frac{\mathrm{d}}{\mathrm{d}t} Z_{u-1}(t_1)$. Then for any $j \in J_0(t_1) \cap \{1, 2, \ldots, u - 1\}$, by the definitions of u and $J_0(\cdot)$, we have $W_j(t_1) = 0$ and $\frac{\mathrm{d}}{\mathrm{d}t} W_j(t_1) > 0$, so there exists $\delta_j > 0$ such that $W_j(t) > 0$ for any $t \in (t_1, t_1 + \delta_j)$. Then following a similar induction argument as the proof for the previous claim that $J(t) \neq \emptyset$, there exists $\delta > 0$ such that $W_j(t) > 0$ for any $t \in (t_1, t_1 + \delta)$ and any $j \in \{1, 2, \ldots, u - 1\}$; i.e., $0 < Z_1(t) < Z_2(t) < \cdots < Z_{u-1}(t)$ for any $t \in (t_1, t_1 + \delta)$. In the actual system with this strict order of queues, either all odd links up to l_u get scheduled at a time slot, or all even links up to l_u get scheduled. Let the service rate on link l_i at time t be $\mu_i(t) = \frac{\mathrm{d}}{\mathrm{d}t} D_i(t)$ for regular time t and any $i \in \{1, 2, \ldots, N\}$. Then in our fluid limits we would have

$$\mu_1(t) = \mu_3(t) = \mu_5(t) = \ldots$$

and

$$\mu_2(t) = \mu_4(t) = \mu_6(t) = \dots$$

up to $\mu_u(t)$ for any regular time $t \in (t_1, t_1 + \delta)$. Then by the absolute continuity,

$$\begin{aligned} D_3(t) - D_1(t) &= D_3(t_1) - D_1(t_1) \\ &\quad + \int_{t_1}^{t} (\mu_3(s) - \mu_1(s)) \mathrm{d}s \\ &= D_3(t_1) - D_1(t_1). \end{aligned}$$

By the definition of derivatives, we have

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}(D_3(t_1) - D_1(t_1)) \\ &= \lim_{t \to t_1^+} \frac{(D_3(t) - D_1(t)) - (D_3(t_1) - D_1(t_1))}{t - t_1} \\ &= 0. \end{aligned}$$

So $\mu_1(t_1) = \mu_3(t_1)$. Similarly, we have

$$\mu_1(t_1) = \mu_3(t_1) = \mu_5(t_1) = \dots$$

and

$$\mu_2(t_1) = \mu_4(t_1) = \mu_6(t_1) = \dots$$

up to $\mu_u(t_1)$. Due to one-hop interference model, we have

$$\mu_1(t_1) + \mu_2(t_1) = 1$$

since at each time slot in the real system either link l_1 or link l_2 must be scheduled. Then by the definition of u, we have $\frac{\mathrm{d}Z_{u-1}}{\mathrm{d}t}(t_1) = \frac{\mathrm{d}Z_u}{\mathrm{d}t}(t_1)$; i.e.,

$$\mu_{u-2}(t_1) + \alpha_{u-1} - \mu_{u-1}(t_1) = \mu_{u-1}(t_1) + \alpha_u - \mu_u(t_1)$$

where $\mu_0(t_1) = 0$ by convention. Then if $u = 2$, we have

$$\begin{aligned} &\begin{cases} \mu_1(t_1) + \mu_2(t_1) = 1 \\ \alpha_1 - \mu_1(t_1) = \mu_1(t_1) + \alpha_2 - \mu_2(t_1) \end{cases} \\ \Rightarrow &\begin{cases} \mu_1(t_1) = \frac{1}{3} + \frac{1}{3}\alpha_1 - \frac{1}{3}\alpha_2 \\ \mu_2(t_1) = \frac{1}{3} - \frac{1}{3}\alpha_1 + \frac{1}{3}\alpha_2 \end{cases} \\ \Rightarrow &\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) = \alpha_1 - \mu_1(t_1) = -\frac{1}{3} + \frac{2}{3}\alpha_1 + \frac{1}{3}\alpha_2. \end{aligned}$$

Similarly, if $u = 3$,

$$\begin{aligned} &\begin{cases} \mu_1(t_1) + \mu_2(t_1) = 1 \\ \mu_1(t_1) = \mu_3(t_1) \\ \mu_1(t_1) + \alpha_2 - \mu_2(t_1) = \mu_2(t_1) + \alpha_3 - \mu_3(t_1) \end{cases} \\ \Rightarrow &\begin{cases} \mu_1(t_1) = \mu_3(t_1) = \frac{1}{2} - \frac{1}{4}\alpha_2 + \frac{1}{4}\alpha_3 \\ \mu_2(t_1) = \frac{1}{2} + \frac{1}{4}\alpha_2 - \frac{1}{4}\alpha_3 \end{cases} \\ \Rightarrow &\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) = \alpha_1 - \mu_1(t_1) = -\frac{1}{2} + \alpha_1 + \frac{1}{4}\alpha_2 - \frac{1}{4}\alpha_3, \end{aligned}$$

and if $u = 4$ we have,

$$\begin{aligned} &\begin{cases} \mu_1(t_1) + \mu_2(t_1) = 1 \\ \mu_1(t_1) = \mu_3(t_1) \\ \mu_2(t_1) = \mu_4(t_1) \\ \mu_2(t_1) + \alpha_3 - \mu_3(t_1) = \mu_3(t_1) + \alpha_4 - \mu_4(t_1) \end{cases} \\ \Rightarrow &\begin{cases} \mu_1(t_1) = \mu_3(t_1) = \frac{1}{2} + \frac{1}{4}\alpha_3 - \frac{1}{4}\alpha_4 \\ \mu_2(t_1) = \mu_4(t_1) = \frac{1}{2} - \frac{1}{4}\alpha_3 + \frac{1}{4}\alpha_4 \end{cases} \\ \Rightarrow &\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) = \alpha_1 - \mu_1(t_1) = -\frac{1}{2} + \alpha_1 - \frac{1}{4}\alpha_3 + \frac{1}{4}\alpha_4. \end{aligned}$$

We can then get the derivative of $Z_1(\cdot)$ at t_1 as

$$\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) = \begin{cases} -\frac{1}{3} + \frac{2}{3}\alpha_1 + \frac{1}{3}\alpha_2 & \text{if } u = 2 \\ -\frac{1}{2} + \alpha_1 + \frac{1}{4}\alpha_{u-1} - \frac{1}{4}\alpha_u & \text{if } u = 3, 5, \dots. \\ -\frac{1}{2} + \alpha_1 - \frac{1}{4}\alpha_{u-1} + \frac{1}{4}\alpha_u & \text{if } u = 4, 6, \dots \end{cases}$$

Since α is admissible, we shall have [1]

$$-R_0^{-1}\alpha < M\gamma \tag{16}$$

for some convex combination coefficients γ, where R_0 is again the matrix consisting of the first N rows of R. Then by (1), we have

$$-R_0^{-1} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & 1 & \ddots & \ddots & 0 \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}.$$

Then by combining the last 2 rows of (16) we have

$$2\alpha_1 + 2\alpha_2 + \cdots + 2\alpha_{N-1} + \alpha_N < 1.$$

So $\frac{\mathrm{d}Z_1}{\mathrm{d}t}(t_1) \leq -\epsilon$, where

$$\begin{aligned} \epsilon = \min \Big\{ &\frac{1}{3} - \frac{2}{3}\alpha_1 - \frac{1}{3}\alpha_2, \\ &\frac{1}{2} - \alpha_1 - \frac{1}{4}\alpha_2 + \frac{1}{4}\alpha_3, \\ &\frac{1}{2} - \alpha_1 + \frac{1}{4}\alpha_3 - \frac{1}{4}\alpha_4, \\ &\dots, \\ &\frac{1}{2} - \alpha_1 - (-1)^{N-1}\alpha_{N-1} - (-1)^N \alpha_N \Big\} \\ > 0. \end{aligned}$$

■

Corollary 1: Given the initial conditions and arrival rates in Lemma 2, there exists $c_2 > 0$ such that $Z_1(t) = 0$ for any $t \geq Z_{\max} c_2$. ⋄

Remark. This comes directly from Lemma 2 and a similar proof of Claim 2 within the proof of Lemma 2. Basically, $Z_1(t)$ has to drop to 0, after which it cannot rise since otherwise the negative derivative forces it to go back to 0.

G. Coupled Network Argument

Based on Corollary 1, we use induction and a coupled network argument to show the following lemma stating the stability of the fluid system, which leads to the stability of the original queueing system by Dai [9].

Lemma 3: Given the initial conditions and arrival rates in Lemma 2, there exists $c_3 > 0$ such that $Z_i(t) = 0$ for any $t \geq Z_{\max} c_3$ and any $i = 1, 2, \ldots, N$. $\diamond$

Proof: We use induction. First, by Corollary 1, there exists $\tilde{c} > 0$ such that $Z_1(t) = 0$ for any $t \geq Z_{\max}\tilde{c}$. Now suppose there exists c and k such that $Z_i(t) = 0$ and $Z_{k+1}(t) > 0$ for any $t \geq Z_{\max} c$ and $i \leq k$. We consider a coupled linear network under the LQF scheduling with $N - k$ links, initial fluids $Z'_i(Z_{\max} c) = Z_{i+k}(Z_{\max} c)$ for $1 \leq i \leq N - k$, and arrival rates

$$\alpha'_1 = \alpha_1 + \alpha_2 + \cdots + \alpha_{k+1}$$

and

$$\alpha'_j = \alpha_{k+j} \qquad j = 2, 3, \ldots, N - k.$$

Thus $(Z'_i(t))_i$ are the fluids of the original network with the first $k + 1$ links combined into one link. Since the fluids satisfy $Z_{k+1}(t) > Z_k(t)$, we have that the queue length at link l_{k+1} is larger than that at link l_k in the actual system. Then by the LQF scheduling, the schedule of the first k links do not affect the schedule of the last $N-k$ links. Also notice that the fluid arrival to $Z_{k+1}(t)$ is $\alpha_1 + \alpha_2 + \cdots + \alpha_{k+1} = \alpha'_1$ since all fluids $Z_i(t)$'s prior to $Z_{k+1}(t)$ remain zero, transferring their exogenous arrival to $Z_{k+1}(t)$. Hence, $Z_{i+k}(t) = Z'_i(t)$ for all $t \geq Z_{\max} c$.

Taking the last $N - k$ rows of (16), we can get

$$-R'^{-1}_0 \alpha' < M'\gamma',$$

where R'_0 consists of the first $N - k$ rows of the routing matrix for the coupled network, M' is the maximal scheduling matrix of the coupled network, and γ' is a set of convex combination coefficients induced from γ. Note that M' is the maximal columns of the matrix formed by the last $N-k$ rows of M. Hence α' is also admissible. Let $Z'_{\max} = \max_i Z'_i(0)$. Then by the Lipschitz continuity we have $Z'_{\max} \leq Z_{\max} c_4$ for some $c_4 > 0$. Again by Corollary 1, there exists $c_5 > 0$ such that $Z'_1(t) = 0$ for any $t \geq Z'_{\max} c_5$. Consequently, $Z_{k+1}(t) = Z'_1(t) = 0$ is also true for $t \geq Z_{\max} c_4 c_5$. By mathematical induction, we get that there exists $c_3 > 0$ such that $Z_i(t) = 0$ for any $t \geq Z_{\max} c_3$ and $1 \leq i \leq N$. ■

IV. Conclusions

We studied the stability of the longest-queue-first scheduling policy in wireless networks with multihop traffic flows and the one-hop interference model. Using fluid techniques, we proved that LQF is throughput optimal in this scenario. The proof itself may be interesting when considering similar fluid systems since we focused on state transition instead of an explicit Lyapunov function. The result may also be a first step to understand the stability performance of LQF in general networks with multihop traffic flows.

Appendix

A. Proof of Proposition 1

First notice $\left|\bar{\mathbb{X}}^{x_k}(0)\right| = \left|\frac{x_k}{|x_k|}\right| = 1$ and $\{x \in \mathcal{X} \mid |x| = 1\}$ is compact, so there exists a subsequence $\left(k_p^{(1)}\right)_p$ such that $\bar{\mathbb{X}}^{x_{k_p^{(1)}}}(0) \to \bar{\mathbb{X}}(0)$ as $p \to \infty$.

By (4) we know for any j, $T_j^{x_{k_p^{(1)}}}(t)$ is Lipschitz continuous with Lipschitz constant 1 and then $\bar{T}_j^{x_{k_p^{(1)}}}(t)$ is also Lipschitz with Lipschitz constant 1. Then the sequence of functions $\left(\bar{T}_j^{x_{k_p^{(1)}}}(t)\right)_p$ is uniformly bounded and equicontinuous on the interval $[0, 1]$ and by the Arzelà-Ascoli theorem there exists a subsequence $\left(k_{1,p}^{(1)}\right)_p$ of $\left(k_p^{(1)}\right)_p$ such that $\left(\bar{T}_j^{x_{k_{1,p}^{(1)}}}(t)\right)_p$ converges on $[0, 1]$ uniformly as $p \to \infty$. Then for the interval $[0, 2]$ there exists a subsequence $\left(k_{2,p}^{(1)}\right)_p$ of $\left(k_{1,p}^{(1)}\right)_p$ such that $\left(\bar{T}_j^{x_{k_{2,p}^{(1)}}}(t)\right)_p$ converges on $[0, 2]$ uniformly as $p \to \infty$. By induction, for any positive integer q, we can find a subsequence $\left(k_{q,p}^{(1)}\right)_p$ such that $\left(\bar{T}_j^{x_{k_{q,p}^{(1)}}}(t)\right)_p$ converges on $[0, q]$ uniformly as $p \to \infty$. We take the diagonal subsequence $\left(k_p^{(2)}\right)_p$ by $k_p^{(2)} = k_{p,p}^{(1)}$ and then $\left(\bar{T}_j^{x_{k_p^{(2)}}}(t)\right)_p$ converges u.o.c. as $p \to \infty$. In the same way we can find a subsequence $(k_p)_p$ such that for any $j = 1, 2, \ldots, r$,

$$\bar{T}_j^{x_{k_p}}(t) \to \bar{T}_j(t) \quad \text{u.o.c. as } p \to \infty.$$

Similarly, $Y_i(\cdot)$ and $D_i(\cdot)$ are Lipschitz with constant 1, so we can find $(k_p)_p$ such that for all $i = 1, 2, \ldots, N$,

$$\bar{Y}_i^{x_{k_p}}(t) \to \bar{Y}_i(t) \quad \text{u.o.c. as } p \to \infty$$

$$\bar{D}_i^{x_{k_p}}(t) \to \bar{D}_i(t) \quad \text{u.o.c. as } p \to \infty.$$

The exogenous arrivals satisfy SLLN, so we may assume for the sample path ω and all $i = 1, 2, \ldots, N$,

$$\frac{1}{|x_{k_p}|} E_i(\lfloor |x_{k_p}| t \rfloor) \to \alpha_i t \quad \text{u.o.c. as } p \to \infty.$$

Then by (2) and (3),

$$\bar{A}_i^{x_{k_p}}(t) \to \bar{A}_i(t) \quad \text{u.o.c. as } p \to \infty$$

$$\bar{Z}_i^{x_{k_p}}(t) \to \bar{Z}_i(t) \quad \text{u.o.c. as } p \to \infty.$$

Then (9), (10), (11), (12), (13) and (15) readily come from (7), (2), (3), (4), (5) and (8).

Notice that (14) is equivalent to the following: Whenever $\bar{Z}_i(t) > 0$, there exists $\delta > 0$ such that $\bar{Y}_i(t') = \bar{Y}_i(t)$ for any $t' \in [t, t + \delta]$. To show this is true, we use a technique from Dai and Prabhakar [11]. We consider a time $t \geq 0$ and suppose $\bar{Z}_i(t) > 0$. Then by continuity there exists $\delta > 0$ such that $\min_{t' \in [t, t+\delta]} \bar{Z}_i(t') > 0$. Set

$a = \min_{t' \in [t, t+\delta]} \bar{Z}_i(t')$. Thus by uniform continuity, there exists $K \geq 0$ such that for any $p \geq K$,

$$\bar{Z}_i^{x_{k_p}}(t') \geq a/2 \quad \forall t' \in [t, t+\delta].$$

Then

$$Z_i^{x_{k_p}}(|x_{k_p}|t') \geq 1 \quad \forall t' \in [t, t+\delta].$$

That is, all systems in the subsequence $(k_p)_p$ have nonempty queue at link l_i during a period of time slots. By the work-conserving property in (6), the cumulative idle time of link l_i can increase by at most 1 (possibly because the queue is emptied at the end of the period of time slots); i.e.,

$$0 \leq Y_i^{x_{k_p}}(|x_{k_p}|t') - Y_i^{x_{k_p}}(|x_{k_p}|t) \leq 1 \quad \forall t' \in [t, t+\delta]$$

$$\Rightarrow 0 \leq \bar{Y}_i^{x_{k_p}}(t') - \bar{Y}_i^{x_{k_p}}(t) \leq \frac{1}{|x_{k_p}|} \quad \forall t' \in [t, t+\delta].$$

Then as $p \to \infty$,

$$\bar{Y}_i(t') = \bar{Y}_i(t) \quad \forall t' \in [t, t+\delta]$$

so we have (14).

Note that by repeatedly taking subsequences we can find $(k_p)_p$ such that all the convergences aforementioned hold at the same time. All components of $\bar{\mathbb{X}}$ are absolutely continuous because they are Lipschitz continuous. The monotonicity of A, D, T, Y implies the monotonicity of $\bar{A}, \bar{D}, \bar{T}, \bar{Y}$.

B. Proof of Lemma 1

We first notice that if $Z \in B$, then there are no adjacent dominating nodes; i.e., if $Z \in B_i$ for some $i \in \{1, 2, \ldots, N\}$, then $Z \notin B_{i-1}$ and $Z \notin B_{i+1}$. Let the dominating set at time t be

$$I_{\text{dom}}(t) = \{i \in \{1, 2, \ldots, N\} \mid Z(t) \in B_i\}.$$

Then we can easily check that $I_{\text{dom}}(t) \subseteq \bigcap \text{LQF}(Z(t))$; i.e., all dominating links must be scheduled by LQF at time t. Due to the one-hop interference, there is no interval arrival to a scheduled link. Then for regular time t and any $i \in I_{\text{dom}}(t)$,

$$\frac{\mathrm{d}A_i}{\mathrm{d}t}(t) = \frac{\mathrm{d}E_i}{\mathrm{d}t}(t) = \alpha_i$$

and

$$\frac{\mathrm{d}D_i}{\mathrm{d}t}(t) = 1,$$

while

$$\frac{\mathrm{d}D_{i-1}}{\mathrm{d}t}(t) = \frac{\mathrm{d}D_{i+1}}{\mathrm{d}t}(t) = 0.$$

Thus,

$$\frac{\mathrm{d}Z_i}{\mathrm{d}t}(t) - \frac{\mathrm{d}Z_{i-1}}{\mathrm{d}t}(t) \leq \alpha_i - 1 \tag{17}$$

and

$$\frac{\mathrm{d}Z_i}{\mathrm{d}t}(t) - \frac{\mathrm{d}Z_{i+1}}{\mathrm{d}t}(t) \leq \alpha_i - 1. \tag{18}$$

We now make two claims to complete the proof of Lemma 1.

Claim 1: There exists $t_1 \in (t_0, t_0 + Z_{\max}/(1-\alpha_i)]$ such that $Z(t_1) \notin B_i$; i.e., link l_i is not dominating anymore at some time before $t_0 + Z_{\max}/(1-\alpha_i)$. ◇

Proof: Indeed, if link l_i remains dominating up to (and including) time $t_0 + Z_{\max}/(1-\alpha_i)$, then by (17), (18) and the absolute continuity of the fluids, we would have for any adjacent link l_j of l_i,

$$\begin{aligned}[Z_i(t_0 + Z_{\max}/(1-\alpha_i)) - Z_j(t_0 + Z_{\max}/(1-\alpha_i))] \\ - [Z_i(t_0) - Z_j(t_0)] \leq -Z_{\max}.\end{aligned}$$

Hence

$$\begin{aligned} &Z_i(t_0 + Z_{\max}/(1-\alpha_i)) - Z_j(t_0 + Z_{\max}/(1-\alpha_i)) \\ &\leq Z_i(t_0) - Z_{\max} \\ &\leq 0. \end{aligned}$$

Then by continuity, there is some $t_1 \in (t_0, t_0 + Z_{\max}/(1-\alpha_i)]$ such that $Z_i(t_1) - Z_j(t_1) = 0$, which contradicts our assumption that link l_i remains dominating up to $t_0 + Z_{\max}/(1-\alpha_i)$. This completes the proof of the claim. ■

Therefore, for any $i \in I_{\text{dom}}(t_0)$, there exists $t_1 \in (t_0, t_0 + Z_{\max}/(1 - \max_j \alpha_j)]$ such that $Z(t_1) \notin B_i$.

Claim 2: If $Z(t_1) \notin B_i$, then $Z(t) \notin B_i$ for any $t \geq t_1$.◇

Proof: Indeed, if $Z(t_2) \in B_i$ for some $t_2 > t_1$, let $t_3 = \sup\{t < t_2 \colon Z(t) \notin B_i\}$. Then by Lipschitz continuity $Z_i(t_3) = Z_j(t_3)$ for some neighbor l_j of link l_i and $t_3 < t_2$. Since $\frac{\mathrm{d}}{\mathrm{d}t}(Z_i(t) - Z_j(t)) \leq \max_k \alpha_k - 1$ for almost all $t \in [t_3, t_2]$, we have

$$Z_i(t_2) - Z_j(t_2) \leq Z_i(t_3) - Z_j(t_3) + (t_2 - t_3)(\max_k \alpha_k - 1) \leq 0,$$

which contradicts the assumption that $Z(t_2) \in B_i$. Hence, $Z(t) \notin B_i$ for any $t \geq t_1$. ■

Considering all i, we have $Z(t) \notin B$ for any $t \geq t_0 + Z_{\max} c_1$, where $c_1 = 1/(1 - \max_j \alpha_j) > 0$.

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